Photonik
Ingenieurbüro
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PE-0700 Abbe Refractometer

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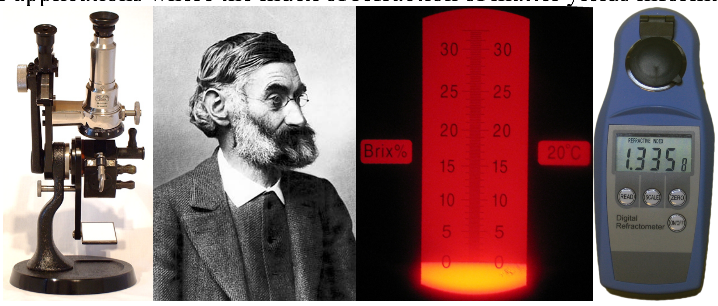
1.0 Abstract

Still, refractometer either as laboratory, handheld or digital model are used in many fields of industry, farming, food processing and the manifold of other applications where the index of refraction of matter yields information of particular properties.



1928 Ernst Abbe (1840-1905 German Physicist) 2009

Although there exists a number of other refractometer, the Abbe refractometer became most popular. Within this experiment the idea of the Abbe refractometer is applied to demonstrate the total reflection and its use to estimate the index of refraction. The set-up demonstrates the effect, however it is not intended to perform precision measurements as known from analytical devices.

2.0 Introduction

The fundamental law which describes the geometrical behaviour of light when passing from one medium to another is defined as the refraction law, stated by Willebrord Snell (Snellius) in the year 1621.

Figure 1.1: Willebrord Snell (1580 - 1626) was a dutch scientist

At this point we will not stress the ancient geometrical optics as used by W. Snellius (1621) rather than using more modern ways of explanation. James Clerk Maxwell (1831 - 1879) and Heinrich Hertz (1857 - 1894) discovered that light shows the properties of electromagnetic waves and therefore can be treated with the theory of electromagnetism, especially with the famous Maxwell equations.

Heinrich Hertz
(1857 - 1894) German

James Clerk Maxwell
(1831 - 1879) Scotsman

2.1 The miracle of the light

Light, the giver of life, has always fascinated human beings. It is therefore natural that people have been trying to find out what light actually is, for a very long time. We can see it, feel its warmth on our skin but we cannot touch it.

The ancient Greek philosophers thought light was an extremely fine kind of dust, originating in a source and covering the bodies it reached. They were convinced that light was made up of particles.

As human knowledge progressed and we began to understand waves and radiation, it was proved that light did not, in fact, consist of particles but that it is an electromagnetic radiation with the same characteristics as radio waves. The only difference is the wavelength.

We now know, that the various characteristics of light are revealed to the observer depending on how he sets up his experiment. If the experimentalist sets up a demonstration apparatus for particles, he will be able to determine the characteristics of light particles. If the apparatus is the one used to show the characteristics of wavelengths, he will see light as a wave.

The question we would like to be answered is: What is light in actual fact? The duality of light can only be understood using modern quantum mechanics. Heisenberg showed, with his famous „Uncertainty relation", that strictly speaking, it is not possible to determine the position x and the impulse p of a particle of any given event at the same time.

$$\Delta x \cdot \Delta p_x \geq \frac{1}{2}\hbar \qquad \textbf{(Eq. 1)}$$

If, for example, the experimentalist chooses a set up to examine particle characteristics, he will have chosen a very small uncertainty of the impulse Δpx. The uncertainty Δx will therefore have to be very large and no information will be given on the location of the event.

Uncertainties are not given by the measuring apparatus, but are of a basic nature. This means that light always has the particular property the experimentalist wants to measure. We determine any characteristic of light as soon as we think of it. Fortunately the results are the same, whether we work with particles or wavelengths, thanks to Einstein and his famous formula:

$$E = m \cdot c^2 = h \cdot \nu \qquad \textbf{(Eq. 2)}$$

This equation states that the product of the mass *m* of a particle with the square of its speed *c* corresponds to its energy

E. It also corresponds to the product of Planck's constant h and its frequency ν, in this case the frequency of luminous radiation.

2.2 Optics and Maxwell's Equations

It seems shooting with a cannon on sparrows if we now introduce Maxwell's equation to derive the reflection and refraction laws. Actually we will not give the entire derivation rather than describe the way. The reason for this is to figure out that the disciplines Optics and Electronics have the same root namely the Maxwell's equations. This is especially true if we are aware that the main job has been done by electrons but it will be done more and more by photons.

Accordingly future telecommunication engineers or technicians will be faced with a new discipline the optoelectronics. We consider now the problem of reflection and other optical phenomena as interaction with light and matter. The key to the description of optical phenomena are the set of the four Maxwell's equations as:

$$\nabla \times \vec{H} = \varepsilon \cdot \varepsilon_0 \cdot \frac{\partial \vec{E}}{\partial t} + \sigma \cdot \vec{E} \text{ and } \nabla \cdot \vec{H} = 0 \quad \textbf{(Eq. 3)}$$

$$\nabla \times \vec{E} = -\mu \cdot \mu_0 \cdot \frac{\partial \vec{H}}{\partial t} \text{ and } \nabla \cdot \vec{E} = \frac{4\pi}{\varepsilon} \cdot \rho \quad \textbf{(Eq. 4)}$$

ε0 is the dielectric constant of the free space. It represents the ratio of unit charge (As) to unit field strength (V/m) and amounts to 8.859 1012 As/Vm.

ε represents the dielectric constant of matter. It characterises the degree of extension of an electric dipole acted on by an external electric field. The dielectric constant ε and the susceptibility χ are linked by the following relation:

$$\varepsilon = \frac{1}{\varepsilon_0} \cdot (\chi + \varepsilon_0) \quad \textbf{(Eq. 5)}$$

$$\varepsilon \cdot \varepsilon_0 \cdot \vec{E} = \vec{D} \quad \textbf{(Eq. 6)}$$

The expression (Eq. 6) is therefore called „dielectric displacement“ or simply displacement.

σ is the electric conductivity of matter.

The expression

$$\sigma \cdot \vec{E} = \vec{j}$$

represents the electric current density

µ0 is the absolute permeability of free space. It gives the relation between the unit of an induced voltage (V) due to the presence of a magnetic field H of units in Am/s. It amounts to 1.256 106 Vs/Am.

µ is like ε a constant of the matter under consideration. It describes the degree of displacement of magnetic dipoles under the action of an external magnetic field. The product of permeability µ and magnetic field strength H is called magnetic induction.

ρ is the charge density. It is the source which generates electric fields. The operation ∇ or div provides the source strength and is a measure for the intensity of the generated electric field. The charge carrier is the electron which has the property of a monopole. On the contrary there are no magnetic monopoles but only dipoles. Therefore $\nabla \cdot \vec{H} = 0$ is always zero.

From (Eq. 3) we recognise what we already know, namely that a curled magnetic field is generated by either a time varying electrical field or a flux of electrons, the principle of electric magnets. On the other hand we see from (Eq. 4). that a curled electromagnetic field is generated if a time varying magnetic field is present, the principle of electrical generator. Within the frame of further considerations we will use glass and air as matter in which the light propagates. Glass has no electric conductivity (e.g. σ =0), no free charge carriers (∇E=0) and no magnetic dipoles (µ = 1). Therefore the Maxwell equations adapted to our problem are as follows:

$$\nabla \times \vec{H} = \varepsilon \cdot \varepsilon_0 \cdot \frac{\partial \vec{E}}{\partial t} \text{ and } \nabla \cdot \vec{H} = 0 \quad \textbf{(Eq. 7)}$$

$$\nabla \times \vec{E} = -\mu_0 \cdot \frac{\partial \vec{H}}{\partial t} \text{ and } \nabla \cdot \vec{E} = 0 \quad \textbf{(Eq. 8)}$$

Using the above equations the goal of the following calculations will be to get an appropriate set of equations describing the propagation of light in glass or similar matter. After this step the boundary conditions will be introduced.

Let's do the first step first and eliminate the magnetic field strength H to get an equation which only contains the electric field strength E.

By forming the time derivation of (Eq. 7) and executing the vector ∇× operation on (Eq. 8) and using the identity for the speed of light in vacuum:

$$c = \frac{1}{\sqrt{\varepsilon_0 \cdot \mu_0}}$$

we get:

$$\Delta \vec{E} - \frac{n^2}{c^2} \cdot \frac{\partial^2 \vec{E}}{\partial t^2} = 0 \quad \textbf{(Eq. 9)}$$

$$\Delta \vec{H} - \frac{n^2}{c^2} \cdot \frac{\partial^2 \vec{H}}{\partial t^2} = 0 \quad \textbf{(Eq. 10)}$$

These are now the general equations to describe the interaction of light and matter in isotropic optical media as glass or similar matter. The Δ sign stands for the Laplace operator which only acts on spatial coordinates:

$$\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

The first step of our considerations has been completed. Both equations contain a term which describes the spatial dependence (Laplace operator) and a term which contains the time dependence. They seem to be very „theoretical“ but their practical value will soon become evident.

2.2.1 Light passing a barrier layer

Now we have to clarify how the wave equations will look like when the light wave hits a boundary. This situation is given whenever two media of different refractive index are in mutual contact. After having performed this step we will be in a position to derive all laws of optics from Maxwell's equations.

Let's return to the boundary problem. This can be solved in different ways. We will go the simple but safe way and request the validity of the law of energy conservation. This

means that the energy which arrives per unit time at one side of the boundary has to leave it at the other side in the same unit of time since there can not be any loss nor accumulation of energy at the boundary.
Till now we did not yet determine the energy of an electromagnetic field. This will be done next for an arbitrary medium. For this we have to modify Maxwell's equations (Eq. 3) and (Eq. 4) a little bit. The equations can be presented in two ways. They describe the state of the vacuum by introducing the electric field strength E and the magnetic field strength H. This description surely gives a sense whenever the light beam propagates within free space. The situation will be different when the light beam propagates in matter. In this case the properties of matter have to be respected. Contrary to vacuum, matter can have electric and magnetic properties. These are the current density j, the displacement D and the magnetic induction B.

$$\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t} + \vec{j} \qquad \textbf{(Eq. 11)}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \textbf{(Eq. 12)}$$

The entire energy of an electromagnetic field can of course be converted into thermal energy δW which has an equivalent amount of electrical energy:

$$\delta W = \vec{j} \cdot \vec{E}$$

From (Eq. 11) and (Eq. 12) we want now to extract an expression for this equation. To do so we are using the vector identity:

$$\nabla \cdot (\vec{E} \times \vec{H}) = \vec{H} \cdot (\nabla \times \vec{E}) - \vec{E} \cdot (\nabla \times \vec{H})$$

and obtaining with (Eq. 11) and (Eq. 12) the result:

$$\delta W = -\nabla \cdot (\vec{E} \times \vec{H}) - \frac{\partial}{\partial t}\left(\frac{\mu\mu_0}{2} \cdot \vec{H}^2 + \frac{\varepsilon \cdot \varepsilon_0}{2} \cdot \vec{E}^2\right)$$

The content of the bracket of the second term we identify as electromagnetically energy Wm

$$W_{em} = \frac{1}{2}\mu\mu_0 \cdot \vec{H}^2 + \frac{1}{2}\varepsilon\varepsilon_0 \cdot \vec{E}^2$$

and the content of the bracket of the first term

$$\vec{S} = \vec{E} \times \vec{H}$$

is known as Poynting vector and describes the energy flux of a propagating wave and is suited to establish the boundary condition because it is required that the energy flux in medium 1 flowing to the boundary is equal to the energy flux in medium 2 flowing away from the boundary. Let's choose as normal of incidence of the boundary the direction of the z-axis of the coordinate system. Than the following must be true:

$$\vec{S}_z^{(1)} = \vec{S}_z^{(2)}$$

$$(\vec{E}^{(1)} \times \vec{H}^{(1)})_z = (\vec{E}^{(2)} \times \vec{H}^{(2)})_z$$

By evaluation of the vector products we get:

$$E_x^{(1)} \cdot H_y^{(1)} - H_x^{(1)} \cdot E_y^{(1)} = E_x^{(2)} \cdot H_y^{(2)} - H_x^{(2)} \cdot E_y^{(2)}$$

Since the continuity of the energy flux must be assured for any type of electromagnetic field we have additionally:

$$E_x^{(1)} = E_x^{(2)} \qquad H_x^{(1)} = H_x^{(2)}$$
$$E_y^{(1)} = E_y^{(2)} \qquad H_y^{(1)} = H_y^{(2)}$$
$$or:$$
$$E_{tg}^{(1)} = E_{tg}^{(2)} \qquad H_{tg}^{(1)} = H_{tg}^{(2)}$$

(The index tg stands for "tangential")
This set of vector components can also be expressed in a more general way:

$$\nabla \times \vec{E} = \vec{N} \times (\vec{E}_2 - \vec{E}_1) = 0$$
$$\text{and} \qquad \textbf{(Eq. 13)}$$
$$\nabla \times \vec{H} = 0$$

$\vec{N}$ is the unit vector and is oriented vertical to the boundary surface. By substituting (Eq. 13) into (Eq. 11) or (Eq. 12) it can be shown that the components of $\vec{B} = \mu\mu_0 \cdot \vec{H}$ and $\vec{D} = \varepsilon\varepsilon_0 \cdot \vec{E}$ in the direction of the normal are continuous, but $\vec{E}$ and $\vec{H}$ are discontinuous in the direction of the normal. Let's summarise the results regarding the behaviour of an electromagnetic field at a boundary:

$$E_{tg}^{(1)} = E_{tg}^{(2)} \qquad D_{norm}^{(1)} = D_{norm}^{(2)}$$
$$H_{tg}^{(1)} = H_{tg}^{(2)} \qquad B_{norm}^{(1)} = B_{norm}^{(2)}$$

By means of the equations (Eq. 11), (Eq. 12) and the above continuity conditions we are now in a position to describe any situation at a boundary. We will carry it out for the simple case of one infinitely spread boundary. This does not mean that we have to take an huge piece of glass, rather it is meant that the dimensions of the boundary area should be very large compared to the wavelength of the light. We are choosing for convenience the coordinates in such a way that the incident light wave (1) is lying within the zx plane Figure 1.2.

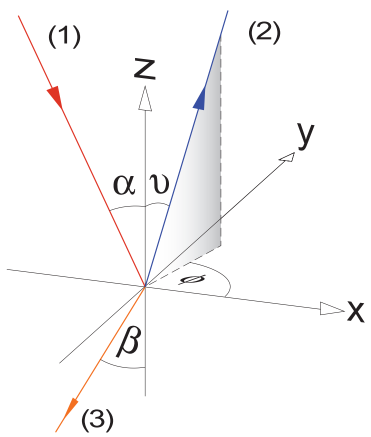


Figure 1.2: Explanation of beam propagation

From our practical experience we know that two additional beams will be present. One reflected (2) and one refracted beam (3). At this point we only define the direction of beam (1) and choosing arbitrary variables for the remaining two. We are using our knowledge to write the common equation for a travelling waves as:

Theory

$$\vec{E}_1 = \vec{A}_1 \cdot e^{-i(\omega_1 \cdot t + \vec{k}_1 \cdot \vec{r}_1)}$$
$$\vec{E}_2 = \vec{A}_2 \cdot e^{-i(\omega_2 \cdot t + \vec{k}_2 \cdot \vec{r} + \delta_2)} \qquad \text{(Eq. 14)}$$
$$\vec{E}_2 = \vec{A}_3 \cdot e^{-i(\omega_3 \cdot t + \vec{k}_3 \cdot \vec{r} + \delta_3)}$$

We recall that A stands for the amplitude, ω for the circular frequency and k represents the wave vector which points into the travelling direction of the wave. It may happen that due to the interaction with the boundary a phase shift δ with respect to the incoming wave may occur. Furthermore we will make use of the relation:

$$\vec{k} = \frac{2 \cdot \pi \cdot n}{\lambda} \cdot \vec{u} = \omega \cdot \frac{n}{c} \cdot \vec{u}$$

whereby n is the index of refraction, c the speed of light in vacuum, l the wavelength, u is the unit vector pointing into the travelling direction of the wave and r is the position vector. Since we already used an angle to define the incident beam we should stay to use polar coordinates. The wave vectors for the three beams will look like:

$$\vec{k}_1 = n_1 \frac{\omega_1}{c} (\sin\alpha, 0, \cos\alpha)$$

$$\vec{k}_2 = n_1 \frac{\omega_2}{c} (\sin\upsilon \times \cos j_2, \sin\upsilon \times \sin j_2, -\cos\upsilon)$$

$$\vec{k}_3 = n_2 \frac{\omega_3}{c} (\sin\beta \times \cos j_3, \sin\beta \times \sin j_3, \cos\beta)$$

Rewriting (Eq. 14) with the above values for the wave vectors results in:

$$\vec{E}_1 = \vec{A}_1 \cdot e^{-i\omega_1\left(t + \frac{n_1}{c}\cdot(x\cdot\sin\alpha + z\cdot\cos\alpha)\right)}$$

$$\vec{E}_2 = \vec{A}_2 \cdot e^{-i\omega_2\left(t + \frac{n_1}{c}\cdot(x\cdot\sin\upsilon\cdot\cos\phi_2 + y\cdot\sin\upsilon\cdot\sin\phi_2 - z\cdot\cos\upsilon)\right) + i\delta_2}$$

$$\vec{E}_2 = \vec{A}_3 \cdot e^{-i\omega_3\left(t + \frac{n_2}{c}\cdot(x\cdot\sin\beta\cdot\cos\phi_3 + y\cdot\sin\beta\cdot\sin\phi_3 - z\cdot\cos\beta)\right) + i\delta_3}$$

2.2.2 The results

Now it is time to fulfil the continuity condition requiring that the x and *y* components of the electrical field E as well as of the magnetic field H are be equal at the boundary plane at *z=0* and for each moment.

$$E_1^x + E_2^x = E_3^x \quad and \quad E_1^y + E_2^y = E_3^y$$
$$H_1^x + H_2^x = H_3^x \quad and \quad H_1^y + H_2^y = H_3^y$$

This is only possible if the all exponents of the set of equations (15) be equal delivering the relation:

$$\omega_1 = \omega_2 = \omega_3$$

Although it seems to be trivial, but the frequency of the light will not be changed by this process. A next result says that the phase shift δ must be zero or π. Furthermore it is required that:

$$\phi_2 = \phi_3 = 0$$

and means that the reflected as well as the refracted beam are propagating in the same plane as the incident beam. A further result is that:

$$\sin\alpha = \sin\upsilon \ \ \text{(law of Reflection)}$$

and finally:

$$n_1 \cdot \sin\alpha = n_2 \sin\beta \qquad or \qquad \frac{\sin\alpha}{\sin\beta} = \frac{n_2}{n_1}$$

the well known law of refraction. We could obtain these results even without actually solving the entire wave equation but rather applying the boundary condition.

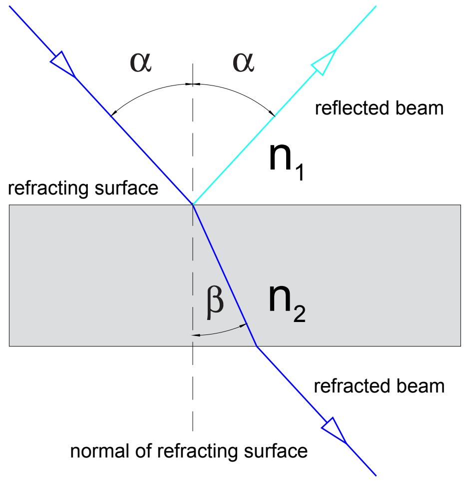


Figure 1.3: Reflection and refraction of a light beam

Concededly it was a long way to obtain these simple results. But on the other hand we are now able to solve optical problems much more easier. This is especially true when we want to know the intensity of the reflected beam. For this case the traditional geometrical consideration will fail and one has to make use of the Maxwell's equations. The main phenomena exploited for the Abbe refractometer is the total reflection at a surface. Without celebrating the entire derivation by solving the wave equation we simply interpret the law of refraction. When we are in a situation where n1 > n2 it may happen that *sin(β)* is required to be *>1*. Since this violates mathematical rules it has been presumed that such a situation will not exist and instead of refraction the total reflection will take place.

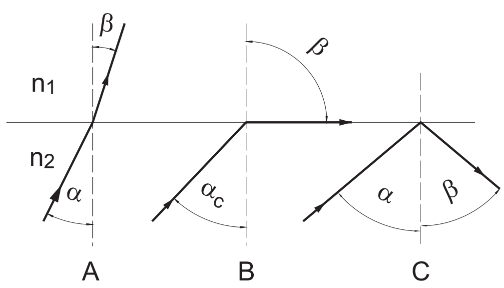


Figure 1.4: From refraction to total reflection

The Figure 1.4 above shows three different cases for the propagation of a light beam from a medium with index of refraction *n2* neighboured to a material with *n1* whereby *n2>n1*. The case A shows the regular behaviour whereas in case B the incident angle reached the critical value of:

$$\sin\beta = \frac{n_2}{n_1} \cdot \sin\alpha_c = 1 \qquad \textbf{(Eq. 15)}$$

The example has been drawn assuming a transition between vacuum (or air) with *n1=1* and BK7 glass with *n2=1.52* (590 nm) yielding the critical value for *αc=41.1°*. Case C shows the situation of total reflection when the value of α > αc and as we know from the law of reflection *α=β*.

3.0 Practical set-up

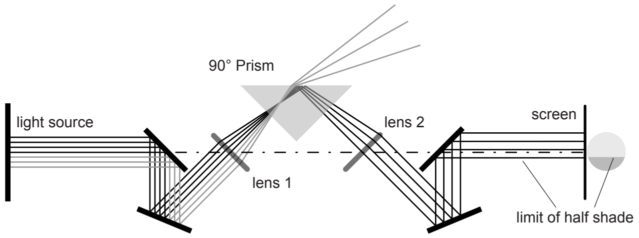


Figure 1.5: Path of rays of the used set-up

Within this experiment we will use a ray path which allows to stay horizontal within the set-up. This requires at least two beam bender as shown in Figure 1.5

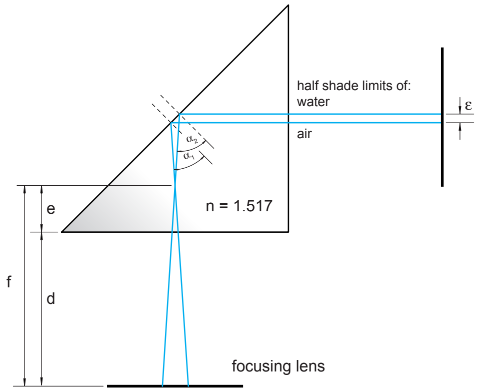


Figure 1.6: Calculation of the movement ε of the halve shade limit

The task of the following considerations is to find the relation which describes the deviation e as function of the index of refraction. For this purpose it is helpful to turn the prism by 45°.

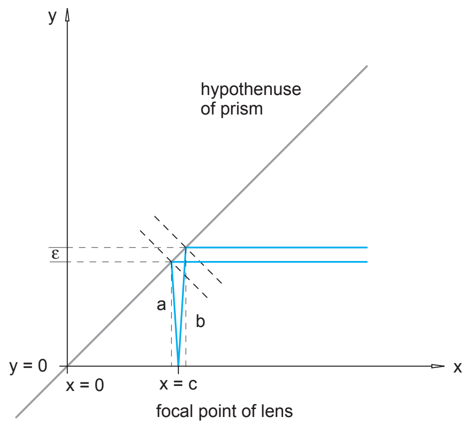


Figure 1.7: The rays a and b are considered as straight line like *y=q+ax*

The ray (a) travels under the critical angle α_1 with respect to the perpendicular of incidence. The index of refraction of the media is air. For case (b) the hypotenuse is covered with water, having an index of refraction of 1.333. Also here the angle of incidence is critical so that this defines the half shade limit in this case. To find the value ε we consider the hypotenuse of the prism as straight line described by the simple formula:

$$y=x \qquad \textbf{(Eq. 16)}$$

Since the slope is 1 (45°) and the coordinate system is chosen in such a way that the intercept is at *y=0 and x=0*. The function of the line (a) is in general:

$$y = y_0 + b_a \cdot x$$

From Figure 1.7 we conclude that for $y = 0$ the value for x is *c*. Consequently y_0 must be

$$y_0 = -b_a c$$

So that the equation for the line (a) becomes:

$$y = -b_a \cdot c + b_a \cdot x \qquad \textbf{(Eq. 17)}$$

We are now interested in the intersection of both curves. In this case equation (Eq. 16) and (Eq. 17) must have the same value for y. Therefore we will get the x value for the intersection of both curves as:

$$x_a = -b_a \cdot c + b_a \cdot x_a$$

or

$$x_a = c\frac{b_a}{b_a - 1}$$

Using (Eq. 16) to obtain the y value of the intersection we find:

$$y_a = c\frac{b_a}{b_a - 1}$$

The same is true for the curve (b)

$$y_b = c\frac{b_b}{b_b - 1}$$

And finally for the value of ε the shift of the half shade limit we get:

$$\varepsilon = y_b - y_a = c\left(\frac{b_b}{b_b - 1} - \frac{b_a}{b_a - 1}\right) \qquad \textbf{(Eq. 18)}$$

From Figure 1.6 and Figure 1.7 we conclude that the value of c is determined as:

$$c = \frac{L}{2} - e$$

Whereby *L* is the length of the cathetus of the right angle prism which is 30 mm in this set-up and the value of *e* is 6 mm resulting in a numerical value for $c = 9\ mm$. Using the known focal length *(f)* and the mechanical distance *(d)* of the lens centre to the cathetus face of the prism the constant c can be redefined to:

$$c=\frac{L}{2}-(f-d)$$

The goal of the next consideration is to connect the slope b of the rays to the respective index of refraction by using (Eq. 15).
In a subsequent step we will find the relation between b and the critical angle α_c. From the mathematics of a curve we know that the slope *b* is related to the angle *b* of the curve by:

$$b=\tan(\beta)$$

From Figure 1.7 we conclude that

$$\beta=225°+\alpha_c$$

Using the 180° periodicity we can rewrite

$$b=\tan(45°+\alpha_c)$$

Applying the angle addition theorem

$$\tan(\alpha+\beta)=\frac{\tan(\alpha)+\tan(\beta)}{1-\tan(\alpha)\cdot\tan(\beta)}$$

We get

$$b=\frac{1+\tan(\alpha_c)}{1-\tan(\alpha_c)}$$

This result is inserted into (Eq. 18) yielding

$$\varepsilon=c\left(\frac{\frac{1+\tan(\alpha_c)}{1-\tan(\alpha_c)}}{\frac{1+\tan(\alpha_c)}{1-\tan(\alpha_c)}-1}-q\right)$$

$$\varepsilon=c\left(\frac{1+\tan(\alpha_c)}{2\tan(\alpha_c)}-q\right)=c\left(\frac{1}{2\tan(\alpha_c)}+\frac{1}{2}-q\right)$$

Now we make use of the relation:

$$\tan(\alpha_c)=\frac{\sin(\alpha_c)}{\sqrt{1-\sin^2(\alpha_c)}}$$

and get

$$\varepsilon=\frac{c}{2}\left(\sqrt{\frac{n_p^2}{n^2}-1}-\sqrt{\frac{n_p^2}{n_{air}^2}-1}\right)=F+\frac{c}{2}\sqrt{\frac{n_p^2}{n^2}-1}$$

$$n=\frac{c\cdot n_p}{\sqrt{4(\varepsilon-F)^2+c^2}} \qquad \textbf{(Eq. 19)}$$

Using the numerical values for the constants as:

$$c=\left(\frac{L}{2}-e\right)=\left(\frac{30mm}{2}-6mm\right)=9mm$$

$$F=-\frac{c}{2}\sqrt{\frac{n_p^2}{n_{air}^2}-1}=-4.5\cdot\sqrt{1.52^2-1}=5.15mm$$

(Eq. 19) becomes to:

$$n=\frac{13.68}{\sqrt{4(\varepsilon-5.15)^2+81}} \qquad \textbf{(Eq. 20)}$$

Note that this equation may slightly differ depending on the parameters of the set-up.

3.1 Experimental Set-Up

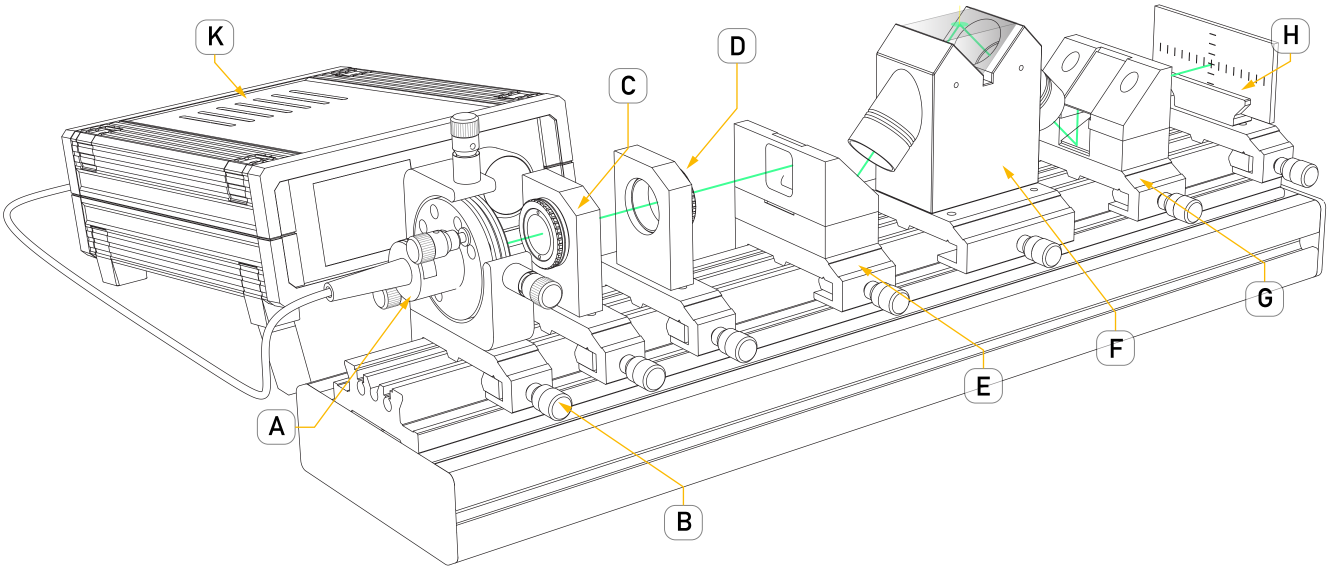
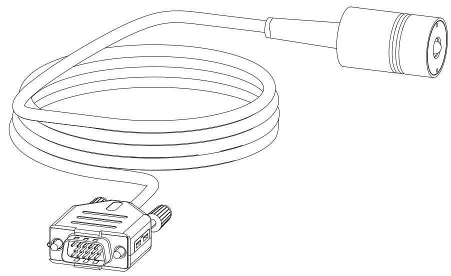


Figure 1.8: Set-up of the refractometer

Description of the components

3.1.1 A. LED light source

The module A consists of a high power LED which emits at 520 nm or 585 nm (optional). The LED is mounted inside a housing with a diameter of 25 mm and fits precisely into the mount of the for axes adjustment holder (B). By means of the four fine pitch adjustment screws and the optical axis of the LED radiation is adjusted with respect to the optical axis of the set-up. Inside the connector an EPROM contains the data of the LED and when connected to the controller, these data are read and displayed by the controller. The laser head is inserted into a rotary stage which is attached to another rotary stage so that the laser head can be rotated around its horizontal and vertical axis.



Module A: LED lamp for 4 axis adjustment holder

3.1.2 K. LED Controller

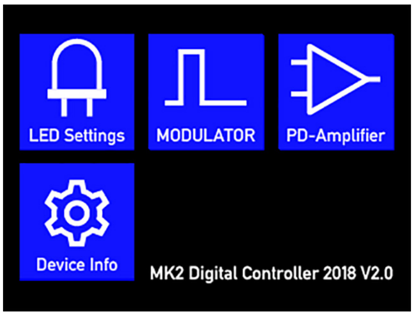
The LED needs a controlled current for its safe operation. For this purpose the power supply is used. Via the 15 pin connector the LED is connected to the rear of device. The power of the entire unit is provided by a 12 V/1A wall plug power supply. This arrangement has the advantage that no mains voltage (240 V) is brought to the experiment.
The desired out power of the LED can be continuously adjusted by means of the central knob after activating the respective settings on the touch panel. When required, the injection current of the LED is modulated and the reference signal for trigger purposes is available via a BNC panel jack at the rear of the device.

Power supply for Module A

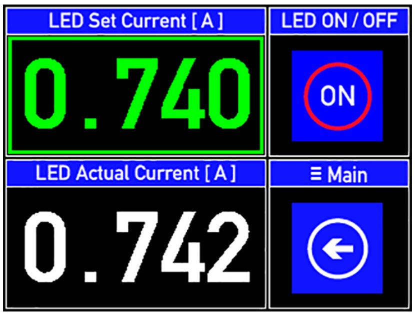
3.1.3 Menu panels of the LED controller

The controller starts with the startup screen. A touch continues to the main menu screen.

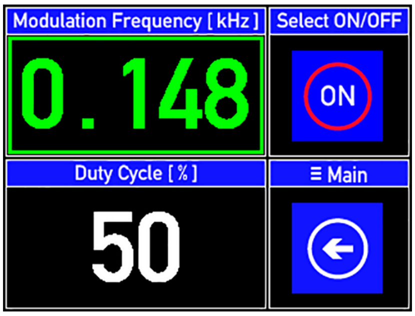
The controller checks whether a light source is connected or not.



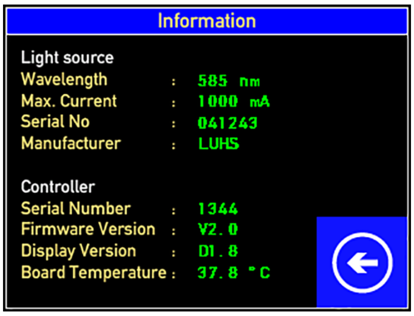
If a light source was detected the main menu appears.



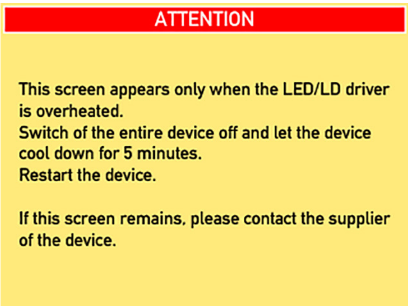
In the LED Settings menu the injection set current is set and the LED switched ON or OFF. In a separate field the actual measure current is displayed.



The Frequency screen allows to set the modulation frequency and to switch the modulation ON or OFF. Depending on the device firmware the duty cycle can be set as well.



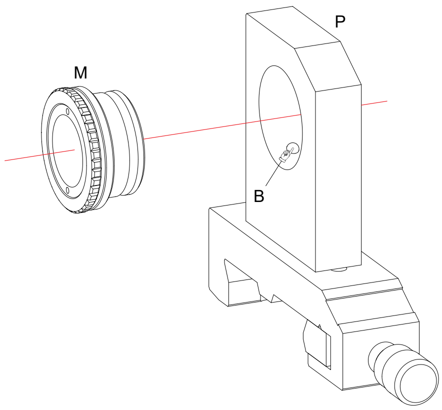
The Device Info shows the parameter of the connected LED as well as the controller's parameter including the internal main board temperature.



In case the LED driver is overheating, a protection circuits switches to the ATTENTION screen.

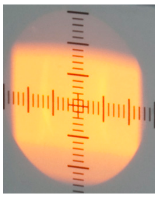
3.1.4 C. Collimating unit

The divergent light of the LED (A) is collimating with an achromatic lens with a focal length of 40 mm. The lens is mounted into a click mount (M). Once inserted into the mounting plate (P) it is kept in position by three spring loaded balls (B).

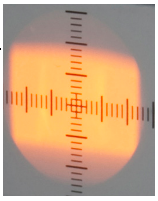


3.1.5 D. Polarising unit

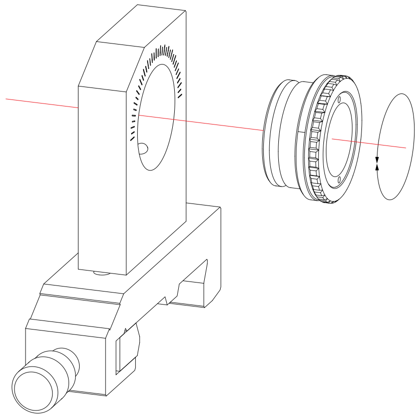
The contrast of the half shade can be effected by different polarisation states of the applied light. To force the LED light into a favourable polarisation a polarizer (1) is provided. By rotating the element the best contrast is adjusted. The polarisation analyser or polarizer has a horizontal rotary stage with a 25 mm through hole bore and 180 degree scale with tick marks for each 5 degrees. It is attached to a 20 mm wide carrier. A film sheet polarizer is inserted into a C25 mount which is set into the rotary stage and is kept with three M3 grub screws in position. The module comes with aligned polarisation direction with 0 degree for vertical polarisation.



The figure on the left shows a usable, however weak contrast of the half shade limit at the bottom of the screen.



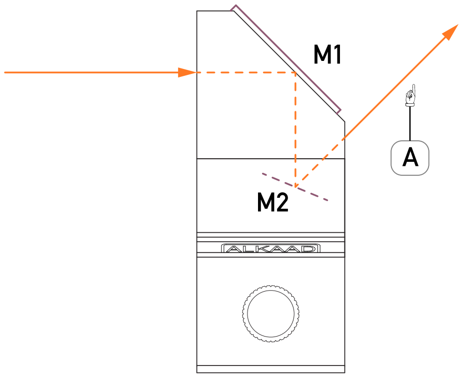
By rotating the polarizer a significant change of the contrast of the half shade is observed.



Module C: Polarising unit

3.1.6 E. Beam bending unit, left 45°

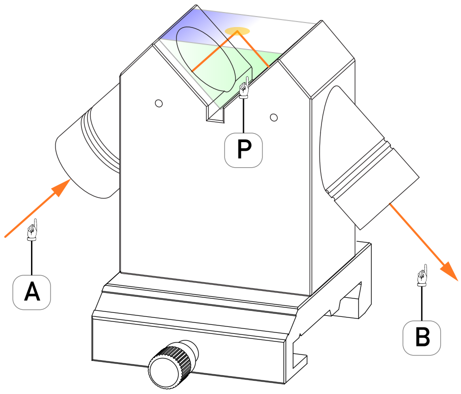
To maintain the optical path as shown in Figure 1.5 the module E is used which bends the beam from the LED by 45° so that the beam enters the first plane of the prism perpendicular. The incident beam first hits the mirror (M1) which is mounted under 45° with respect to the optical axis of the set-up. Subsequently it is reflected by the mirror (M2) which is aligned to 22.5° towards the prism. The entire unit is attached to a 30 mm wide carrier.



Module C: Beam bending unit, left 45°

3.1.7 F. Prism unit with lenses

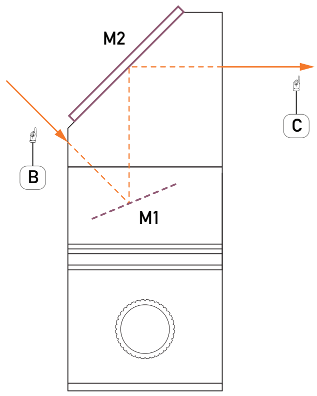
The heart of the set-up forms the prism assembly. A rectangular prism (P) made out of BK7 glass is cemented to the block holder.
The incident beam (A) is focussed by means of a lens inside the adjustable tube and generates a bunch of rays propagating under different angles inside the prism. When it hits the top surface of the prism a couple of rays are transmitted while others are reflected. All those beams having an angle less the critical angle for total reflection are leaving the prism assembly via the right sleeve. Inside of the sleeve another lens is located to provide a sharp image on the provided screen.



Module E: Prism unit with lenses

3.1.8 G. Beam bending unit, right 45°

This module directs the beam (B) from the prism parallel to the optical axis towards the white screen. The incident (B) hits first the mirror (M1) which is mounted under 22.5° with respect to the optical axis of the set-up and subsequently it is reflected by the mirror (M2) which is aligned under 45°.

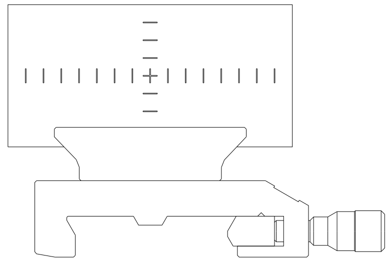


Module F: Beam bending unit, right 45°

Experimental Setup

3.1.9 G. Screen with mm scale

Finally the rays which have been reflected by the upper prism surface are imaged to the white screen. The screen has a horizontal and vertical scale divided in millimetre.
The screen itself is kept by means of M3 grub screws inside the holder allowing the screen to be used in different orientations.



Module F: Screen with horizontal and vertical scale

3.1.10 Set of test liquids

A set of 3 test liquids filled into 15 ml dropping bottles with isopropyl alcohol (IPA), distilled water and a 60 °Bx (Degrees Brix) sugar solution is provided. Brix is a definition of the content of the dry matter in a solution. One degree Brix is 1 gram of sucrose in 100 grams of solution. Other sugar solutions can be made very simple, the corresponding index of refractions are:

Brix	Index of refraction
10	1.344
20	1.364
30	1.381
40	1.400
50	1.420

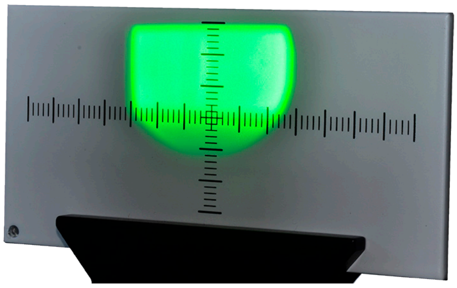
The index of refraction of the provided liquids are:

Water	1.33
IPA	1.38
60 Brix sugar solution	1.44

4.0 Measurements

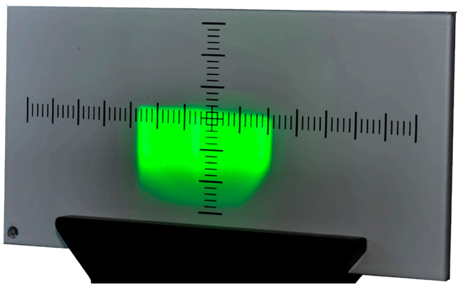
4.1 Zero Measurement against air

This measurement has been performed with the green LED with a peak emission at 520 nm



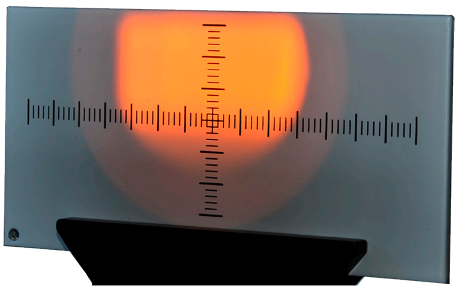
4.2 Measurement with water

The half shade moved by 16 mm



4.3 Zero Measurement against air

This measurement has been performed with the yellow LED with a peak emission at 585 nm



4.4 Measurement with water

The half shade moved by 20 mm.
Entering the measured value into (Eq. 20) the related index of refraction can be calculated and if necessary the instrument factors adapted.